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RESEARCH MEMORANDUM

PRELIMINARY ANALYSIS OF PROBLEM OF DETERMINING
EXPERIMENTAL PERFORMANCE OF AIR-COOLED TURBINE

II - METHODS FOR DETERMINING COOLING-AIR-FLOW
CHARACTERISTICS

By Herman H. Ellerbrock, Jr.

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II - METHODS FOR DETERMINING COOLING-AIR-FLOW CHARACTERISTICS

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SUMMARY

The problem of investigating air-cooled turbine performance is being analyzed. Suggested methods of investigating and analyzing data of such turbines so that performance characteristics can be obtained are being developed. Such an evaluation of a turbine requires knowledge of the flow characteristics of the cooling air in the blade cooling-air passages as the air proceeds from the roots to the tips of the blades. In the investigation of a turbine, certain data must therefore be obtained and formulas developed to obtain these flow characteristics.

The methods that are expected to permit the determination of pressure, temperature, and velocity through the blade cooling-air passages from specific investigations that must be conducted are presented. Formulas are suggested relating these characteristics to certain parameters. The formulas must be verified and the coefficients appearing therein evaluated by investigations of cooled turbines. Discussions of work that leads to the recommended formulas are given in some cases.

INTRODUCTION

Because very little material is available on the methods of investigating air-cooled turbines and analyzing the data obtained to determine their performance characteristics, a study of such methods is being made at the NACA Lewis laboratory. The current formulas for the heat-transfer aspects of the air-cooled turbine, which must be evaluated through investigation of the turbine, are presented in reference 1.

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The evaluation of the cooling and performance characteristics of a turbine (and of an engine in which the turbine may be used) with blades through which cooling air is circulated requires knowledge of the characteristics of the cooling air; that is, pressure, temperature, and velocity as the air proceeds from the roots to the tips of the blades. These characteristics are required in the determination of such factors as the heat flow, blade temperatures, blade-to-cooling-air heat-transfer coefficients, and effective cooling-air temperatures. These factors are discussed in reference 1. In addition, the cooling-air characteristics are required to evaluate the work of pumping the air through the blades and the work required in the compressor to compress the air to the pressure required at the blade roots. The turbine-pumping work is usually insufficient to raise the air pressure from atmospheric to that required at the blade tip to permit the air to flow out of the blade and mix with the combustion gas stream. The air must consequently be raised above atmospheric pressure at the blade root by some means external to the turbine, for example, by the compressor. If the required pressure is known, it is possible to determine the bleedoff point on the compressor. In the investigation of a turbine, certain data must be obtained and formulas developed so that evaluations of the type discussed can be made for various flight conditions.

Formulas for determining the cooling-air-flow characteristics in the turbine-blade cooling-air passages are suggested herein. The formulas must be verified and the coefficients appearing therein evaluated by investigation of cooled turbines. Discussions of work that leads to the suggested formulas are given in some cases, so that readers unfamiliar with the subject may obtain some background knowledge.

ANALYSIS OF FORMULAS FOR COOLING-AIR-FLOW CHARACTERISTICS

Mach Number in Blade Cooling-Air Passage

An analysis of the cooling-air flow through turbine blades based on one-dimensional flow has been developed (reference 2). The work in general is based on the analysis of Shapiro and Hawthorne (reference 3); the main additions of reference 2 to this work is the evaluation of the generalized body force in the equations of reference 3 for the case of a rotating body and the method of numerically solving the equations. From this analysis, the variation of Mach number of the air through the turbine-blade passage was determined as

$$\frac{dM_a^2}{dy} = \frac{I_T}{T_a''} \frac{dT_a''}{dy} - I_F \left[\frac{4f_0 b}{D_{h,a}} - \frac{1 + \frac{\gamma_a - 1}{2} M_a^2}{\gamma_a M_a^2 T_a''} \frac{2\omega^2 r_h b}{g R_a} \left(1 + \frac{b}{r_h} - y \frac{b}{r_h} \right) \right] + I_A \frac{1}{A_a} \frac{dA_a}{dy} \quad (1)$$

(All symbols used herein are defined in appendix A.) The terms I_T , I_F , and I_A are influence coefficients equal to

$$I_T = \frac{M_a^2 (1 + \gamma_a M_a^2) \left(1 + \frac{\gamma_a - 1}{2} M_a^2 \right)}{1 - M_a^2} \quad (2)$$

$$I_F = \frac{\gamma_a M_a^4 \left(1 + \frac{\gamma_a - 1}{2} M_a^2 \right)}{1 - M_a^2} \quad (3)$$

$$I_A = \frac{-2M_a^2 \left(1 + \frac{\gamma_a - 1}{2} M_a^2 \right)}{1 - M_a^2} \quad (4)$$

For a constant-area passage A_a spanwise, the last term of equation (1) drops out.

In order to solve equation (1), the friction factor must be known so that f_0 can be determined for any given conditions. Current theory is inadequate for determining f_0 for the conditions of flow as they exist in the rotating-blade coolant passages; therefore the equation for the friction factor must be established from investigations of turbines. The steps involved are: (1) Obtain measurements of pressures and temperatures in the turbine-blade cooling-air passages; (2) use these data and equation (1) to calculate an average f_0 for the flow conditions imposed; and (3) determine the empirical equation that applies to these experimental values. The details are discussed in the following section.

Method of Obtaining Friction Factor

In a typical turbine, pressures and temperatures of the cooling air in the blade cooling-air passage are difficult to measure. If these pressures and temperature were measured along the blade span, some of the numerical integration procedure for solving equation (1), which is subsequently discussed, would be eliminated; but it is assumed that, except for special turbine rigs, the maximum instrumentation in the blade passages is limited to thermocouples and pressure tubes at the blade roots and tips.

It is therefore assumed that data are obtained from measurements at these positions such that the static and total pressures and the static and total temperatures at these positions are available for a range of cooling-air flows, a range of cooling-air temperatures and pressures, and as wide a range as possible of the ratio of average blade-wall temperature to average cooling-air static temperature. From these pressures and temperatures, the velocities and the Mach numbers at the blade root and tip are calculated.

When the cooling-air conditions at the blade tip for a given flow and so forth are known, equation (1) must be integrated stepwise (using about five steps) from tip to root. A convenient method of numerical integration is given in reference 4. When the cooling-air Mach number and static temperature at the tip are known, influence coefficients can be calculated from equations (2) to (4) or obtained from tables in reference 2. If five steps of the integration process are used, y will then equal the values 0, 0.2, 0.4, 0.6, 0.8, and 1.0. For the first step from $y = 0$ to $y = 0.2$, the area change dA_a/dy of the air passage can be determined from the geometry. In equation (1) for the first step, $y = 0$ and the values of T''_a and A_a are those at the tip. The term dT''_a/dy is determined from the equation (reference 2)

$$\frac{dT''_a}{dy} = K_1 T''_a + \frac{b^2 \omega^2}{J g c_{p,a}} y - \frac{b^2 \omega^2}{J g c_{p,a}} - K_1 T_{g,e} \quad (5)$$

where

$$K_1 = \frac{H_o b l_o}{c_{p,a} w_a (1 + \lambda)} \quad (6)$$

and where

- λ $H_0 l_0 / H_1 l_1$ (H_1 becomes H_f if fins are used in blade passage. See reference 1 for relation between H_1 and H_f .)
- l_0 average outside perimeter of blade, (ft)
- l_1 average inside perimeter of blade, (ft)
- H_0 average coefficient for convection heat transfer from combustion gas to blades, (Btu/(sq ft)(°F)(sec))
- H_1 average coefficient for convection heat transfer from blades to cooling air based on blade-wall area and blade temperatures, (Btu/(sq ft)(°F)(sec))

In the determination of dT''_a/dy from equation (5) for the first step, $y = 0$ and T''_a is the value at the tip. The investigation to get the heat-transfer data can be made at the same time as the cooling-air-flow experiments so that H_0 , H_1 , or H_f , and $T_{g,e}$, values that are calculated as shown by methods in reference 1, can be used in the foregoing equations.

All of the terms for solving equation (1) for dM_a^2/dy for the first step from $y = 0$ to $y = 0.2$ are now known except the friction factor f_0 , which is the term being sought. As subsequently discussed in greater detail, the friction factor is expected to vary along the passage from tip to root; however, the formulas for the variation cannot be obtained in turbine experiments. With the type of instrumentation possible in turbines, only an average value for the passage and a formula for expressing this average value can be obtained. A value for the average friction factor for the entire passage from tip to root is consequently assumed and inserted in equation (1), which is then solved for dM_a^2/dy for the first step.

The next step is to obtain the Mach number at $y = 0.8$ from

$$(M_a^2)_{y=0.2} = (M_a^2)_{y=0} + 0.2 \left(\frac{dM_a^2}{dy} \right)_{y=0} \quad (7)$$

When M_a is known at $y = 0.2$, the influence coefficients are determined again using this Mach number, the area change from $y = 0.2$ to $y = 0.4$ is determined, A_a at $y = 0.2$ is found, and T''_a is calculated from the equation (equation (40), reference 2)

$$T''_a = Ke^{(K_1 y)} + K_2 y + K_3 + T_{g,e} \quad (8)$$

where

$$K = e^{-K_1} (T''_{a,h} - T_{g,e} e^{-K_2 - K_3}) \quad (9)$$

$$K_2 = - \frac{\omega^2 b w_a (1 + \lambda)}{J g H_o l_o} \quad (10)$$

$$K_3 = \frac{\omega^2 w_a (1 + \lambda) (b + r_h)}{J g H_o l_o} - \frac{c_{p,a} \omega^2 w_a^2 (1 + \lambda)^2}{J g H_o^2 l_o^2} \quad (11)$$

$T''_{a,h}$ total temperature of cooling air relative to blade at root (obtained from measurements), $^{\circ}R$

If $c_{p,a}$ is based on the average static temperature of the cooling air obtained from the measurements at blade root and tip, K_1 , K_2 , and K_3 remain unchanged from tip to root. With T''_a known, dT''_a/dy is calculated from equation (5). With these values and the assumed value of friction factor, dM_a^2/dy can again be calculated for $y = 0.2$ from equation (1). The detailed procedure to follow until the conditions at the root are obtained is explained in reference 2.

The values of cooling-air conditions obtained at the root by the procedure described using the assumed value of average friction factor are compared with the measured values. If these values disagree, a new average friction factor is assumed until agreement occurs. This friction factor then applies to the test conditions. This method is the only way known for evaluating f_0 from the experiments. A direct solution of f_0 from equation (1) as yet has not been obtained. The method given is less tedious than it appears because so many terms in the equations remain constant. It should be pointed out that changes in f_0 do not appreciably affect dM_a^2/dy . Small errors in data may consequently cause large variations in the value of f_0 obtained from the experiments in the manner described.

The rigorous method of solving equations (5), (6), and (8) to (11) is to use local values of the factors involved instead of

average values at each point along the blade span considered. The rigorous treatment is described in reference 2 and comparison of results of numerical examples using the rigorous and less-rigorous methods are described therein. Little error results by the use of average values.

Friction factors can thus be determined from an investigation for the ranges of conditions that are described. An equation must be determined that will represent these factors, or in other words, correlate the results.

Friction-Factor Correlation

Before proceeding to a discussion of methods of correlating the experimental friction factors for turbine-blade coolant passages, a brief discussion of knowledge obtained on friction factors inside pipes and other configurations is given. Such background knowledge provides suggestions for possible methods of determining an empirical formula or data correlation from which the blade friction factors can be determined for conditions other than those investigated.

When fluid enters a tube, the flow pattern or velocity profile downstream of the tube inlet continues to change as the fluid proceeds along the tube until, when the distance from the tube inlet has become sufficiently great, about 50 tube diameters, the flow pattern tends to stabilize. Downstream of this point the velocity profile remains unaltered. Upstream of the point where the velocity profile becomes fixed, the friction factor varies appreciably; downstream of this point, the friction factor is independent of the distance from the tube inlet. There is evidence that the point at which the velocity profile becomes fixed depends on the Reynolds number of the flow, the initial turbulence in the air stream approaching the tube inlet, and the type of tube inlet.

The values of friction factor in the region with fixed velocity profile, for either laminar or turbulent flow in this region, have been established with great accuracy by many investigators. For the inlet region where the profile is varying, only scant data are available on the friction-factor variations along the pipe length. Reference 5 gives a historical background on the subject and reports further investigations. The results of reference 5 indicate that a great deal of work remains to be done before adequate formulas for determining friction factors in the tube inlet are obtained.

For isothermal fully developed incompressible turbulent flow in smooth pipes over a limited range of Reynolds numbers from 5000 to 200,000, the following equation well represents the great amount of data that has been accumulated (reference 6, p. 119):

$$f_0 = \frac{0.046}{(\text{Re})^{0.2}} \quad (12)$$

With the use of the equations for velocity distribution with fully developed turbulent flow, von Kármán (reference 7) predicts the following relation between f_0 and Re for smooth pipes:

$$\frac{1}{\sqrt{4f_0}} = -0.8 + 2 \log_{10} \text{Re} \sqrt{4f_0} \quad (13)$$

which fits the experimental data for friction in smooth pipes and is recommended for extrapolation to high values of Re .

For isothermal laminar or streamline flow in a straight circular pipe for fully developed laminar velocity profile, the pressure drop due to friction is given by Poiseuille's law from which the friction factor becomes

$$f_0 = \frac{16}{\text{Re}} \quad (14)$$

In equations (12) to (14), the Reynolds number is

$$\text{Re} = \frac{\rho V D}{\mu} \quad (15)$$

and the friction factor is defined as

$$f_0 = \frac{\Delta p_f}{\frac{1}{2} \rho V^2} \frac{D}{4L} \quad (16)$$

which definition gives an "apparent" friction factor. (For details, see reference 5.)

The foregoing discussion of correlation formulas for pipes has been for isothermal flow. In accordance with reference 6, for the case of heating or cooling inside pipes the viscosity term in

Re should be based on a special temperature and then the same formulas for f_0 in isothermal flow can be used for nonisothermal flow. For Re below 2100, this temperature is

$$T = \text{average static temperature of fluid} + \frac{\text{average surface temperature of pipe} - \text{average static temperature of fluid}}{4} \quad (17)$$

Above a Re of 2100, the same formula is used except that the constant 4 is changed to 2.

In an investigation on heat transfer and flow in pipes with very high surface temperatures (reference 8), there is evidence that both ρ and μ should be based on the pipe surface temperature for satisfactory correlation of the friction factor. This phenomenon is similar to that occurring in the correlation of the heat-transfer coefficients discussed in reference 1. In addition, from reference 8 it is evident that ρ in the definition of f_0 (equation (16)) should be based on the pipe-surface conditions. Some analysis of the data of reference 9 on flow in pipes with ratios of surface temperature to stream temperature ranging in some cases higher than 2 again showed evidence that for correlation purposes both ρ and μ in Reynolds number should be based on surface temperature. The analysis is quite incomplete and is only reported as a guide.

As in the case of Reynolds number determination for heat-transfer coefficients (reference 1), if the passage is of non-circular cross section the hydraulic diameter of the passage D_h is used as the dimension D in the Reynolds number for evaluating f_0 . The hydraulic diameter is equal to four times the cross-sectional area of the passage divided by the wetted perimeter (fig. 1).

For tubes of annular cross section, the hydraulic diameter is equal to the diameter of the outer pipe minus the diameter of the inner pipe. Over a wide range of Reynolds numbers with fully developed turbulent velocity profile with such cross sections, a formula similar to equation (12) is applicable, the exponent of the Reynolds number being 0.2 as in equation (12). For tubes of annular cross section, a wide dissimilarity of results exists, however, for the value of the constant in equation (12). In

reference 6 (p. 123) the circular-pipe value of 0.046 is used and in reference 10 a slightly lower value of 0.044 is suggested. In references 11 and 12 a much higher value is advocated. In reference 11, for instance, it was determined from data that the value 0.046 of circular pipes should be increased by multiplying by the factor

$$\xi' = \frac{\frac{D_o}{D_i} + \xi}{\frac{D_o}{D_i} + 1} \quad (18)$$

where ξ equals a function of the pipe diameters that is given in reference 11 as

$$\xi = \frac{2 \log_e \frac{D_o}{D_i} - \left(\frac{D_o}{D_i}\right)^2 + 1}{\frac{D_o}{D_i} - \frac{D_i}{D_o} - 2 \frac{D_o}{D_i} \log_e \frac{D_o}{D_i}} \quad (19)$$

where

D_o diameter of outside pipe

D_i diameter of inside pipe

Many of the methods of friction-factor correlation in the foregoing discussion can be applied to turbine-blade coolant passages. In practice it is expected, for instance, that the passages will always be noncircular, varying from approximately the blade shape for a hollow thin-walled blade to an annular section for the case of a hollow blade with an insert. The treatment of such shapes and status of correlating friction-factor data for them were consequently brought out in the foregoing discussion. The turbine-blade cooling-air passages can be expected to have inlet shapes at the blade root varying from turbine to turbine and different length-to-diameter ratios. The length-to-diameter ratios will probably in most cases be less than that required to set up a fixed velocity profile. It can therefore be expected that the friction factor will vary along the passage and that the average value for the blades of one turbine may be quite different from that of another turbine. The establishment of the laws or formulas for evaluating the friction factor spanwise and as a function

of Reynolds number for wide ranges of ratio of blade temperature to stream temperature, flow rate, and so forth would require a vast amount of instrumentation in the passages. Such instrumentation would not only be impractical to install but would clutter up the blade passages of commercial turbines in such a manner as to possibly make the measurements inaccurate. Such laws must consequently be established on large turbine rigs with large blades. Such a rig for air-cooled turbine-blade research is now being fabricated at the Lewis laboratory.

The average friction factors for a given blade configuration determined by methods given in the previous section from measured data at the blade root and tip reflect the effects of the inlet shape, conditions upstream of the blade root, and the changing velocity profile as the air proceeds through the blade. For the purpose of a turbine experiment, it is recommended that, for correlating purposes, these friction factors be plotted against the values of a Reynolds number of the cooling-air flow. Whether the data for all the experiments will correlate on one curve depends in great measure on the method for calculating the Reynolds number of the cooling-air flow. It is recommended as a first trial that the Reynolds number be calculated in the same manner as given in reference 1 for evaluating the convection heat-transfer coefficient from blade to cooling air. In reference 1, it is recommended that the density and the viscosity in the Reynolds number be based on conditions at the blade surface.

In calculating the velocity to use in the Reynolds number, which is the average relative stream velocity W_a as brought out in reference 1, the average density of the fluid stream in the cooling passages of the blades is required. This density in turn requires knowledge of the average static pressure and temperature in the cooling passages. The methods of determining this pressure and temperature are given in the following section.

Friction-factor data established and correlated as described serve two purposes: First, the data make possible the evaluation of performance of the turbine on which they were established; and second, the data provide a means of checking general laws or formulas established on large rigs. From these general formulas, it should be possible to calculate the variation of friction factor of the cooling passage spanwise along the blade for specific conditions and blade geometry of a given turbine, to integrate these local friction factors to obtain the average friction factor, and then to compare them with the data obtained on the turbine as described herein.

The basic equation (equation (1)) is based on one-dimensional flow, as pointed out, and as a consequence has limitations. Some effects, such as Coriolis effects, may invalidate the analysis to some extent but at present the equation is the best that is available. This equation is better than most equations that are obtained for similar flow problems where important assumptions are made that would cause great error in the results for conditions as they exist in a turbine.

Solutions for Flow Characteristics

From measurements at the blade root and tip in the cooling-air passage and the cooling-air flow, all characteristics, static and total pressures, static and total temperatures, velocity, and Mach number at each of these two positions, can be obtained. It was assumed that such measurements would be made.

In the integration of equation (1) stepwise for the case where with a certain assumed friction factor the calculated Mach number at the root equaled the measured Mach number, both total temperature and Mach number of the cooling air at various stations, $y = 1, 0.8, 0.6, 0.4, 0.2$, and 0 , become available. Because of the verification of experimental root Mach number with the assumed f_0 , these total temperatures and Mach numbers are true values and can be used to calculate the other flow characteristics at these stations.

When M_a and T''_a are known at each station, the static temperature can be calculated from

$$T_a = \frac{T''_a}{1 + \frac{\gamma_a - 1}{2} M_a^2} \quad (20)$$

The ratio of specific heats γ_a should be based on T_a . At low Mach numbers, T''_a can be used. At high Mach numbers, T''_a can be used first; then when T_a is calculated, a correction can be made to γ_a and T_a can be recalculated.

The velocity at each station can be obtained from

$$W_a = \sqrt{M_a^2 \gamma_a g R_a T_a} \quad (21)$$

The static pressure at each station can be calculated from a combination of the perfect gas and continuity equations, or

$$p_a = \frac{w_a R_a T_a}{W_a A_a} \quad (22)$$

Finally, the total pressure at each station is obtained from

$$p''_a = p_a \left(\frac{T''_a}{T_a} \right)^{\frac{\gamma_a}{\gamma_a - 1}} \quad (23)$$

When average pressures, temperatures, or velocities are required, it is best to plot the values at each station against the distance from the root to the station involved, to integrate the curve with a planimeter, and then to divide by the distance from root to tip. If the curves are practically straight lines, such detail is not required, but cases may occur where the variation of a characteristic may be far from linear and errors result by arithmetically averaging either the root and tip measured values or the calculated station values.

SUMMARY OF METHOD

The formulas for determining the cooling-air-flow characteristics of an air-cooled turbine are summarized as follows.

The variation of the cooling-air Mach number in the passage is determined from equation (1) and the influence coefficients therein are determined using equations (2) to (4).

The last term of equation (1) is eliminated when a cooling passage of constant cross section is considered. Inasmuch as measurements are made at the root and the tip of the blade cooling-air passage, the actual Mach number at these points can be calculated. Various average friction factors are assumed for equation (1) and the Mach number at the root is determined and compared with the value calculated from the pressure and temperature measurements. The value of assumed friction factor that causes agreement is the experimental value to use.

In the integration of equation (1), about five steps are used, at $y = 0, 0.2, 0.4, 0.6, 0.8$, and 1.0 . The change in area from step to step is determined from the geometry of the blade and dT''_a/dy can be found from equation (5). At the tip, the T''_a value to use in equation (1) is the measured value and $y = 0$. For $y = 0.2$ and so forth, equation (8) is used to determine T''_a . The procedure for determining dM_a^2/dy for the various y positions is given in reference 2.

For isothermal turbulent flow in smooth pipes, the correlation equation for the friction factor is given by equation (12) and for laminar flow, by equation (14). The Reynolds number is defined by equation (15), density and viscosity being based on free-stream temperature. The friction factor based on free-stream conditions is defined by equation (16) and, as before, the density is based on free-stream conditions. In the case of pipes where either heating or cooling is encountered, the viscosity term in the Reynolds number of the equation of laminar flow (equation (14)) is based on a temperature defined by equation (17). For a Reynolds number greater than 2100, the constant 4 in equation (17) is changed to 2.

Some references indicate that for annular passages the constant in equation (12) is much higher than 0.046 and should be increased by multiplying by the factor ξ' defined by equation (18). The factor ξ appearing in equation (18) is defined by equation (19).

As discussed in the analysis, there is evidence that the friction-factor data can only be correlated by basing the viscosity and the density on blade-surface temperatures rather than the free-stream static conditions when the ratio of surface temperature to stream temperature is much different from 1.

When the Mach number at the root has been calculated through use of equation (1) and equals the measured Mach number, the total temperatures and Mach numbers at the various y positions chosen are known. With these known factors and by means of equations (20) to (23), the other flow conditions at these positions can be calculated.

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APPENDIX - SYMBOLS

The following symbols are used in this report:

- A cross-sectional area, sq ft
- b blade height or span, ft
- c_p specific heat at constant pressure, Btu/(lb)(°F)
- D diameter, ft
- D_h hydraulic diameter $4A/l$, ft
- f_0 apparent friction factor based on stream conditions outside boundary layer (See equation (16).)
- g ratio of gravitational force to absolute unit of mass, lb/slug, or acceleration due to gravity, ft/sec²
- H average convection heat-transfer coefficient, Btu/(sq ft)(°F)(sec)
- I_A influence coefficient,
$$\frac{-2M_a^2 \left(1 + \frac{\gamma_a - 1}{2} M_a^2 \right)}{1 - M_a^2}$$
- I_F influence coefficient,
$$\frac{\gamma_a M_a^4 \left(1 + \frac{\gamma_a - 1}{2} M_a^2 \right)}{1 - M_a^2}$$
- I_T influence coefficient,
$$\frac{M_a^2 (1 + \gamma_a M_a^2) \left(1 + \frac{\gamma_a - 1}{2} M_a^2 \right)}{1 - M_a^2}$$
- J mechanical equivalent of heat, 778.3 ft-lb/Btu
- $K = e^{-K_1} (T''_{a,h} - T_{g,e} - K_2 - K_3)$
- $K_1 = \frac{H_o b l_o}{c_{p,a} w_a (1 + \lambda)}$

$$K_2 = - \frac{\omega^2 b w_a (1 + \lambda)}{J g H_o l_o}$$

$$K_3 = \frac{\omega^2 w_a (1 + \lambda) (b + r_h)}{J g H_o l_o} - \frac{c_{p,a} \omega^2 w_a^2 (1 + \lambda)^2}{J g H_o^2 l_o^2}$$

L pipe length, ft

l perimeter, ft

M Mach number

p static pressure, lb/sq ft absolute

p" total pressure relative to moving blade, lb/sq ft absolute

Δp_f friction pressure drop, lb(force)/sq ft

R gas constant, ft-lb/(lb)(°F)

Re Reynolds number (See equation (15).)

r radius, ft

T static temperature, °R

T" total temperature relative to moving blade, °R

V absolute velocity, ft/sec

W velocity relative to moving blade, ft/sec

w weight flow, lb/sec

$$y = \frac{r_T - r_x}{b}$$

γ ratio of specific heats

$$\lambda = \frac{H_o l_o}{H_1 l_1} \quad (H_1 \text{ becomes } H_f \text{ if fins are used in blade passage. See reference 1 for relation between } H_1 \text{ and } H_f.)$$

- μ absolute viscosity, slugs/(sec)(ft)
- ρ density, slugs/cu ft
- ξ function of pipe diameter (See equation (19).)

$$\xi = \frac{\frac{D_o}{D_i} + \xi}{\frac{D_o}{D_i} + 1}$$

- ω angular velocity, radians/sec

Subscripts:

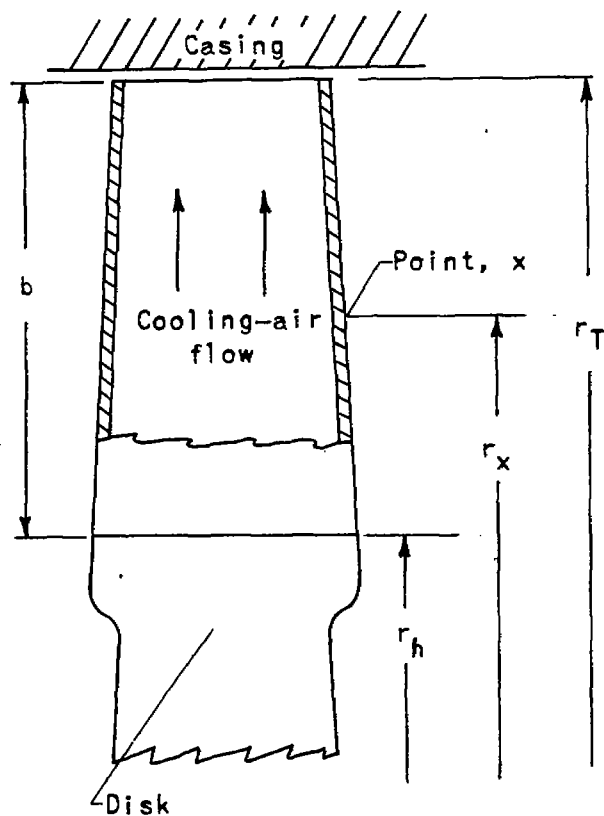
- O stream conditions outside of boundary layer
- a cooling air
- e effective, used with symbol for temperature and denotes temperature effecting heat transfer
- f finned blades, or friction
- g combustion gas
- h blade root
- i inside
- o outside
- T blade tip
- x point along blade span

Figure 1 shows the dimensions of a hollow blade as used in this analysis.

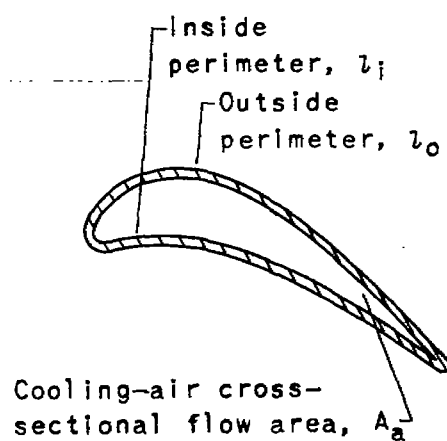
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(a) Elevation of hollow blade.



(b) Cross section of typical hollow blade.



Figure 1. - Sketches showing dimensions of typical hollow air-cooled turbine blade.

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